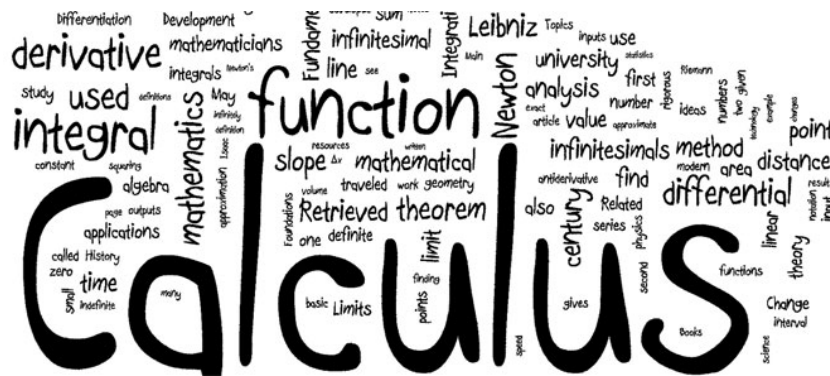


Monmouth Regional High School

Calculus Honors

2021 Summer Assignment



This packet is the summer assignment for students entering Calculus Honors. This will be collected on the first day of class. On the first day of class, be prepared for a quiz on any of these topics as well as the Unit Circle. You will be expected and required to be highly proficient in working with trigonometry without using cheat sheets, charts and calculators. No excuses.

YOU MUST SHOW ALL OF YOUR WORK TO RECEIVE FULL CREDIT!

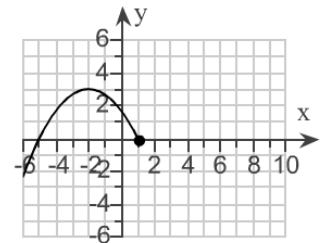
Name: _____

Welcome to Calculus Honors!

- The math tasks in this packet cover a variety of skills and concepts that we will use throughout Calculus Honors. It is *required* that you know and understand all of these concepts thoroughly, before you enter this class.
- Work on this packet must be clear and neat.
- All work must be shown for each problem.
- The work for this packet is due on the first day of class. No excuses will be accepted.

Show all work for each problem.

1. Use the graph of f to determine each of the following. Where applicable, use interval notation.



- (a) the zero(s) of f
- (b) $f(-4.5)$
- (c) the value of x for which $f(x) = 3$
- (d) Is $f(-1)$ positive or negative?

a) Zeros: _____

b) $f(-4.5) =$ _____

c) $x =$ _____

d) Circle one: Positive or negative

2.

The domain of the piecewise function is $(-\infty, \infty)$.

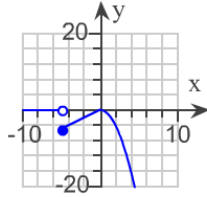
a. Graph the function.

b. Use your graph to determine the function's range.

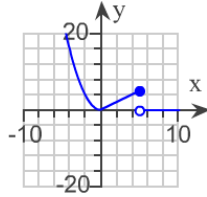
$$f(x) = \begin{cases} 0 & \text{if } x < -5 \\ -x & \text{if } -5 \leq x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

a. Choose the correct graph below.

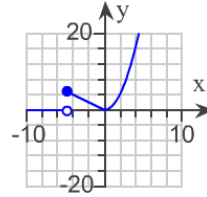
A.



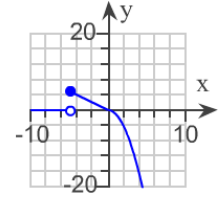
B.



C.



D.

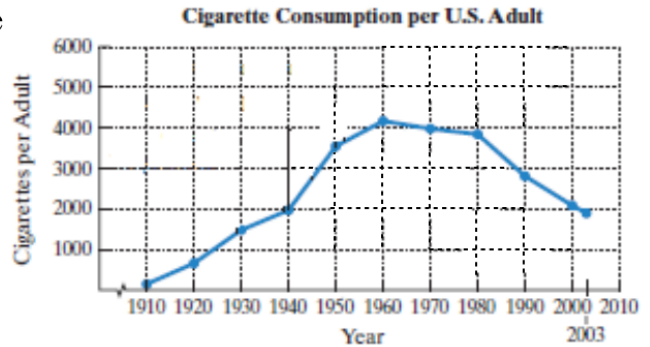


b. Range: _____

3.

The data in the graph can be modeled by the following piecewise function where x represents years after 1910 and $f(x)$ represents cigarette consumption per U.S. adult.

$$f(x) = \begin{cases} 61.9x + 132 & \text{if } 0 \leq x \leq 30 \\ -2.2x^2 + 256x - 3503 & \text{if } 30 < x \leq 93 \end{cases}$$



Use the piecewise function to find the cigarette consumption in 2003. How well does the function describe the actual consumption for the year shown by the line graph?

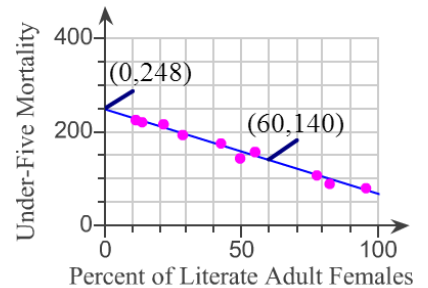
What was the cigarette consumption in 2003? _____

Does the function accurately describe the actual cigarette consumption in 2003?

Yes

No

4. Shown to the right is a scatter plot that indicates a relationship between the percentage of adult females in a country who are literate and the mortality of children under five (per thousand). Also shown is a line that passes through or near the points. Complete parts a through d.



a. According to the graph, what is the y-intercept of the line? Describe what this represents in this situation.

b. Use the coordinates of the two points shown to compute the slope of the line. Describe what this means in terms of the rate of change.

Slope: _____

Fill in the blanks: For each 1% of adult females who are literate, the mortality rate of children under five _____ by _____ per thousand.

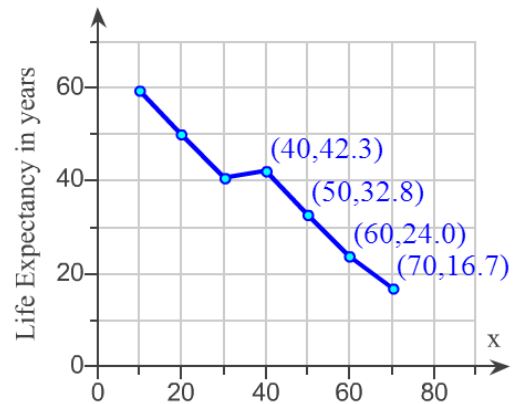
c. Use the y-intercept from part (a) and the slope from part (b) to write a linear function that models child mortality, $f(x)$, per thousand, for children under five in a country where $x\%$ of adult women are literate.

d. Use the function from part (c) to predict the mortality rate of children under five in a country where 50% of adult females are literate.

5.

The graph on the right shows the remaining life expectancy, E , in years for females of age x .

Find the average rate of change between the ages of 50 and 60.
Describe what the average rate of change means in this situation.



Average rate of change:

Describe situation:

If the average rate of change **between ages 70 and 80** is -0.44 , this means that

- A. life expectancy decreases by 0.44 years between the ages of 70 and 80.
- B. life expectancy increases by 0.44 years between the ages of 70 and 80.
- C. life expectancy is 4.4 years between ages 70 and 80.
- D. life expectancy decreases each year between the ages of 70 and 80 by 0.44 years.

6. Suppose that a ball is rolling down a ramp. The distance traveled by the ball is given by the function $s(t) = 4t^2$, where t is the time, in seconds, after the ball is released, and $s(t)$ is measured in feet. Find the ball's average velocity in each of the following time intervals.

a. $t_1 = 3$ to $t_2 = 4$

b. $t_1 = 3$ to $t_2 = 3.5$

c. $t_1 = 3$ to $t_2 = 3.01$

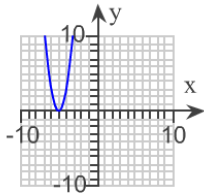
7.

Begin by graphing the standard quadratic function, $f(x) = x^2$. Then use transformations of this graph to determine the graph of the given function.

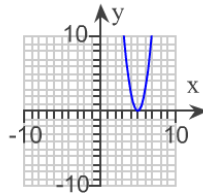
$$g(x) = \frac{1}{3}(x - 5)^2$$

Choose the correct graph of $g(x)$ below.

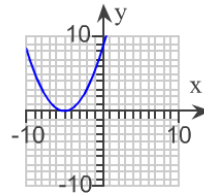
A.



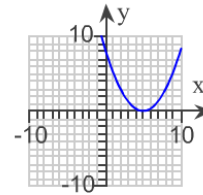
B.



C.

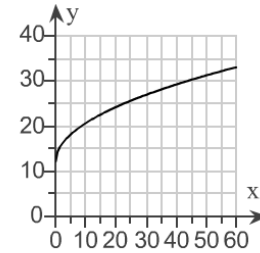


D.



8.

The function $g(x) = 2.7\sqrt{x} + 12.1$ models the median height, $g(x)$, in inches, of children who are x months of age. The graph of g is shown.



a. Describe how the graph can be obtained using transformations of the square root function $f(x) = \sqrt{x}$.

- A. Stretch $f(x)$ horizontally by a factor of 2.7. Shift $f(x)$ right by 12.1 units.
- B. Shrink $f(x)$ horizontally by a factor of 2.7. Shift $f(x)$ down by 12.1 units.
- C. Stretch $f(x)$ vertically by a factor of 2.7. Shift $f(x)$ up by 12.1 units.
- D. Stretch $f(x)$ horizontally by a factor of 2.7. Shift $f(x)$ up by 12.1 units.

b. According to the model, what is the median height of children who are 24 months, or 2 years, old? Use a calculator to find the median height.

The actual median height for children at 24 months is 25 inches. How well does the model describe the actual height?

The model describes the actual height A. very well.
 B. poorly.

c. Use the model to find the average rate of change, in inches per month, between birth and 6 months.

d. Use the model to find the average rate of change, in inches per month, between 30 and 36 months.

How does this compare with your answer in part (c)? How is this difference shown by the graph?

- A. The average rate of change is larger. The graph is steeper.
- B. The average rate of change is smaller. The graph is steeper.
- C. The average rate of change is larger. The graph is not as steep.
- D. The average rate of change is smaller. The graph is not as steep.

9. Given the function $f(x) = x^3 - 1$,

(a) Find $f^{-1}(x)$.

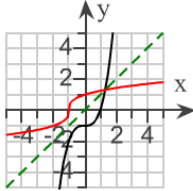
(b) Graph f and f^{-1} in the same rectangular coordinate system.

(c) Use interval notation to give the domain and the range of f and f^{-1} .

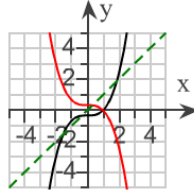
(a) Find $f^{-1}(x)$.

(b) Choose the correct graph which shows f and f^{-1} graphed in the same coordinate system.

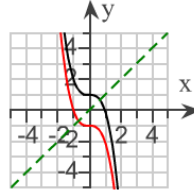
A.



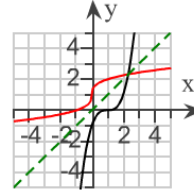
B.



C.



D.



(c) State the domain and range of f and f^{-1} using interval notation.

10.

Graph the rational function. Answer parts **1.** through **7.** below.

$$f(x) = \frac{6}{x^2 - x - 2}$$

1. Select the symmetry of the function.

- The function is symmetric about the y-axis.
- The function is symmetric about the origin.
- The function has no symmetry about the y-axis or the origin.

2. Find the y-intercept. Select the correct choice below and fill in any answer boxes within your choice.

- A. The y-intercept is .
- B. There is no y-intercept.

3. Find any x-intercepts.

4. What are the x-coordinates of the vertical asymptote(s)?

5. What is the y-coordinate of the horizontal asymptote?

6. State the y-coordinate for each of the following points on the function:

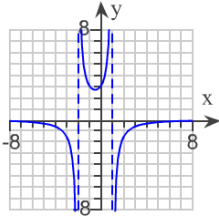
(-5,)

(0,)

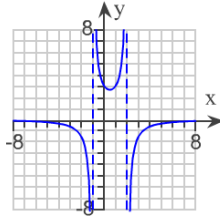
(4,)

7. Using the information determined above, select the graph of the rational function.

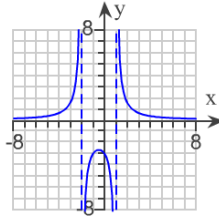
A.



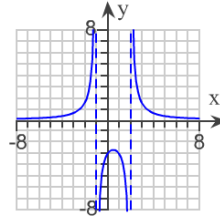
B.



C.



D.



11. Find the domain of the following logarithmic function.

$$f(x) = \ln(x - 4)^2$$

Which is the correct domain of $f(x) = \ln(x - 4)^2$?

$(-\infty, -4)$ or $(-4, \infty)$

$(-\infty, 4)$ or $(4, \infty)$

$(4, \infty)$

$(-\infty, \infty)$

12. The loudness level of a sound, D , in decibels, is given by the formula $D = 10 \log(10^{12}I)$, where I is the intensity of the sound, in watts per meter². Decibel levels range from 0, a barely audible sound, to 160, a sound resulting in a ruptured eardrum. The sound of a certain animal can be heard 500 miles away, reaching an intensity of 1.7×10^3 watts per meter². Determine the decibel level of this sound. At close range, can the sound of this animal rupture the human eardrum?

Decibel level:

At close range can the sound rupture the human eardrum?

13. Without using a calculator, determine the exact value of each expression.

You will be expected to know the unit circle when class starts.

a. $\sin \pi$

b. $\cos \frac{\pi}{4}$

c. $\sin \frac{5\pi}{6}$

d. $\cot \frac{3\pi}{4}$

e. $\sec \frac{2\pi}{3}$

f. $\csc \frac{5\pi}{3}$

g. $\tan 2\pi$

h. $\csc \frac{\pi}{6}$

i. $\sec \frac{5\pi}{6}$

j. $\sin^{-1} \left(\tan \frac{3\pi}{4} \right)$

k. $\left(\cos \frac{\pi}{6} \right)$

l. $\sin \frac{17\pi}{3}$

Simplify

14. $\frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$

15. $\frac{4 - \frac{12}{2x-3}}{5 + \frac{15}{2x-3}}$

Solve

16. $\sin^2 x = \frac{1}{2}$

17. $2 \cos^2 x - 1 - \cos x = 0$

18. Find $[g \cdot h](x)$ and $[h \cdot g](x)$ for $g(x) = 7x$ and $h(x) = -5x^3 + 9x^2 - 2x + 2$.

19. Simplify $\frac{x^{\frac{4}{7}} \cdot x^{\frac{3}{7}}}{x^{\frac{1}{7}}}$.

20. Prove/Verify the following:

a. $\cos x + \sin x \tan x = \sec x$

b. $\frac{1}{\tan x} + \tan x = \frac{\sec^2 x}{\tan x}$

c. $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$